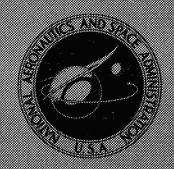
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STRAIN-RATE EQUATIONS FOR CALCULATION OF SECONDARY CREEP DEFORMATION OF THICK-WALLED TUBES WITH INTERNAL OR EXTERNAL PRESSURE

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Tubular structures such as heat exchangers and fuel-pin cladding may be operated at tempera-					
tures and stress levels such that creep occurs during normal operation for long lifetimes.					
Equations are presented relating the strain rate for secondary creep in thick-walled tubes at con-					
stant temperature with static internal pressure. Stress equations and strain-rate equations are					
given for the compressive secondary creep of thick-walled tubes at constant temperature with					
constant external pressure. The equations given are applicable in designing tubes undergoing					
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STRAIN-RATE EQUATIONS FOR CALCULATION OF SECONDARY CREEP DEFORMATION OF THICK-WALLED TUBES WITH INTERNAL OR EXTERNAL PRESSURE

by Richard E. Morris

Lewis Research Center

SUMMARY

The design of tubular components of nuclear propulsion systems for long service life at high temperature and pressure requires that the secondary creep deformation be controlled or limited. Examples of such structures include heat exchangers and nuclear-reactor fuel-pin cladding.

Heat exchangers must operate with internal pressure at high temperature for long lifetimes. An important requirement is the limitation of the creep expansion of the thick-walled heat-exchanger tubing.

In the case of nuclear-reactor fuel-pin cladding, both external coolant pressure and the development of internal fission gas pressure cause creep deformations that must be limited.

Strain-rate equations are presented for use in these design problems. The stress and strain-rate relations apply to thick-walled tubes for both internal and external pressure. Both temperature and pressure were assumed to be constant. Stress and strain rate were assumed to be related by a power law which includes a function of temperature as a parameter. Thus, the equations are applicable to the stress and strain-rate behavior of tubes over a range of temperatures.

INTRODUCTION

Tubular components of nuclear propulsion systems may be designed to operate under extreme conditions of temperature and pressure such that creep deformation occurs continuously during operation.

An important limitation in designing heat exchangers for high-temperature operation is the creep strength of heat-exchanger materials. Similarly, the creep strength of nuclear-reactor fuel-pin cladding materials poses a limitation on the operating temperature of the fuel pins in a reactor.

Helium-to-air heat exchangers may operate with an internal helium pressure of 7 to 14 meganewtons per square meter at temperatures from 1000 to 1150 K for lifetimes of 50 000 hours. Under these conditions the heat-exchanger tubes undergo continuous creep deformation. Rupture of heat-exchanger tubes must be avoided. The amount of deformation accumulated during the operating lifetime of the tubing must be limited by design.

Tubular nuclear-reactor fuel-pin cladding contains the nuclear fuel. Fission gases are generated as the fuel is consumed. Gases collect in the high-temperature core void of a fuel pin. The gas is constrained by the volume of void provided in the fuel-pin design so that pressure develops as a function of fuel burnup. Pressures as high as 25 meganewtons per square meter may be generated during the life of a pin.

For high-performance reactors the peak temperature of fuel-pin cladding may be from 1100 to 1500 K. In this temperature range, the regularly increasing fission gas pressure will cause the cladding to creep during the 10 000-hour design life that is typical of these fuel pins. Radioactive fission gases must be contained within the fuel pin. Rupture must be avoided. Total creep deformation during the lifetime of the fuel-pin cladding must be limited by design.

The coolant in the gas-cooled reactor must operate under pressure sufficient to obtain the heat-transfer characteristics needed to cool the fuel pins. For helium, a coolant pressure of about 10 meganewtons per square meter is needed. This pressure is sufficient to cause continuous compressive creep of the fuel-pin cladding during the first part of the life of the reactor. Compressive creep must be limited to avoid collapse of fuel-pin cladding. The fission gas pressure gradually increases to oppose the coolant pressure. Fission gas pressure is greater than the coolant pressure at the end of the life of the reactor.

Each of these design problems involves the relation between stress and strain rate in thick-walled tubes undergoing creep from either external or internal pressure.

This report presents stress equations and strain-rate equations for applications to these problems. The analysis follows the method presented by Bailey (ref. 1) which is a three-dimensional stress analysis of internally pressurized tubes undergoing secondary creep. The analysis presented in this report changes one of his fundamental assumptions so that the strain-rate equations apply to a range of operating temperatures. In addition, by using the boundary conditions for external pressure the analysis yields stress equations and strain-rate equations for tubes under external pressure. All of the equations apply only to the case of constant pressure and temperature.

SYMBOLS

- A material constant, $hr^{-1}(N/m^2)^{-n}$
- a inside radius, cm
- B material constant, $hr^{-1} (N/m^2)^{-n}$
- b outside radius, cm
- C constant of integration
- D constant of integration
- ΔH apparent activation energy, J/(K)(mole)
- n stress exponent
- p pressure, N/m²
- R gas constant, 8.3143 J/(K)(mole)
- r radius, cm
- T absolute temperature, K
- u radial displacement, cm
- $\dot{\epsilon}$ creep rate, hr⁻¹
- $\frac{\cdot}{\epsilon}$ equivalent creep rate, hr⁻¹
- ρ b/a
- σ stress, N/m²
- $\overline{\sigma}$ equivalent stress, N/m²
- χ b/r

Subscripts:

- i internal
- o external
- r radial
- z axial
- θ circumferential

ANALYSIS

This analysis follows the work of Bailey for the three-dimensional stress analysis of internally pressurized tubes (ref. 1) except for the power law assumption that relates stress and strain rate. Bailey assumed a power law $\dot{\epsilon} = A \overline{\sigma}^n$. In this form both A and n are temperature dependent. Consequently, strain-rate relations for creep in thick-walled tubes under internal pressure based on this power law are applicable only for a relatively narrow range of temperature for which A and n are constant.

The power law assumed in this analysis contains a temperature correction term $e^{-\Delta H/RT}$. Use of this term reduces the temperature dependence of the constants in the stress, creep-strain-rate relation. With the temperature correction term included, the creep-strain-rate relation used in this analysis is

$$\frac{\dot{\epsilon}}{\dot{\epsilon}} = B \overline{\sigma}^n e^{-\Delta H/RT}$$

This power law has been used by several investigators, for example, Maag and Mattson (ref. 2). Garofalo (ref. 3) indicates that this strain-rate relation applies to secondary creep at constant temperature for many metals and alloys at low stresses. Long lifetimes with low creep rates at high temperature limit the use of many metals to low stresses.

The use of the power law with the temperature correction term in this analysis and adding the boundary conditions for tubes with external pressure provide strain-rate relations for thick-walled tubes loaded with static internal or external pressure undergoing isothermal secondary creep as a function of temperatures.

The assumptions used in the analysis are as follows:

- (1) The tube material is isotropic.
- (2) Creep associated with the deformation of the tubes is secondary creep.
- (3) Secondary creep strain follows the von Mises flow rule.
- (4) Stress and strain rate are assumed to be related by the empirical power law $\dot{\epsilon} = \pm B \overline{\sigma}^n e^{-\Delta H/RT}$. (The sign is selected for tensile (+) or compressive (-) strain rate.)
- (5) The axial creep rate is assumed to be zero. Planes perpendicular to the axis of the tube remain plane. The creep deformation is one of plane strain.
- (6) The principal axes of stress and creep strain rate are coincident and remain so during strain up to moderate amounts.
 - (7) The principal shear strain rates are proportional to the principal shear stresses.
- (8) Poisson's ratio is assumed to be 1/2. This assumption requires that the sum of principal orthogonal creep rates equal zero, that is $\dot{\epsilon}_r + \dot{\epsilon}_{\theta} + \dot{\epsilon}_z = 0$.

INTERNAL PRESSURE

The use of the power law assumption with the temperature correction factor results in strain-rate equations for $\dot{\epsilon}_{\theta}$ and $\dot{\epsilon}_{r}$. Correlation of the equations with experimental data will provide values for the material constants. The resulting equations are more general than Bailey's strain-rate equations, in that the equations presented here include temperature as a parameter.

The stress equations derived by Bailey (ref. 1) are not affected by this change in the stress-strain rate power law assumption. The equations for the stresses in a tube undergoing secondary creep at constant temperature are given here in the terminology of this report for completeness (the derivation is included in the appendix):

$$\sigma_{\mathbf{r}} = -\frac{\chi^{2/n} - 1}{\rho^{2/n} - 1} p_{\mathbf{i}} \tag{1}$$

$$\sigma_{\theta} = \frac{1 - \left(\frac{n-2}{n}\right) \chi^{2/n}}{\rho^{2/n} - 1} p_{i}$$
(2)

$$\sigma_{z} = \frac{1 - \left(\frac{n-1}{n}\right) \chi^{2/n}}{\rho^{2/n} - 1} p_{i}$$
(3)

$$\overline{\sigma} = \frac{\sqrt{3}}{n} \frac{\chi^{2/n}}{\rho^{2/n} - 1} p_{i} \tag{4}$$

Equation (4) is obtained by substituting the stress equations into the relation for the maximum distortion energy stress (equivalent stress)

$$\overline{\sigma} = \frac{1}{\sqrt{2}} \left[(\sigma_{\mathbf{r}} - \sigma_{\theta})^2 + (\sigma_{\theta} - \sigma_{\mathbf{z}})^2 + (\sigma_{\mathbf{z}} - \sigma_{\mathbf{r}})^2 \right]^{1/2}$$

Use of the modified power law assumption yields the following strain-rate equations:

$$\dot{\epsilon}_{\mathbf{r}} = -\frac{\sqrt{3}}{2} \, \mathbf{B} \overline{\sigma}^{\mathbf{n}} \mathbf{e}^{-\Delta \mathbf{H}/\mathbf{R}\mathbf{T}} \tag{5}$$

$$\dot{\epsilon}_{\theta} = \frac{\sqrt{3}}{2} \, \text{B} \bar{\sigma}^{\text{n}} \text{e}^{-\Delta H/\text{RT}} \tag{6}$$

$$\epsilon_{Z} = 0$$
 (assumption (5)) (7)

$$\frac{\dot{\epsilon}}{\epsilon} = B\overline{\sigma}^{n} e^{-\Delta H/RT}$$
 (assumption (4), tensile creep rate) (8)

These eight equations define the stresses and strain rates in an internally pressurized tube at constant temperature for secondary creep conditions.

External Pressure

The derivation for stresses in tubes from internal pressure uses the boundary conditions

$$\sigma_{\mathbf{r}} = -\mathbf{p}$$
 at $\mathbf{r} = \mathbf{a}$

$$\sigma_{\mathbf{r}} = 0$$
 at $\mathbf{r} = \mathbf{b}$

Changing the boundary conditions to

$$\sigma_{\mathbf{r}} = 0$$
 at $\mathbf{r} = \mathbf{a}$

$$\sigma_{\mathbf{r}} = -\mathbf{p}$$
 at $\mathbf{r} = \mathbf{b}$

we have the condition for tubes loaded by external pressure. Then if the boundary conditions are used to evaluate the constants of integration in the derivation, the equations for the stresses become

$$\sigma_{\mathbf{r}} = -\frac{\rho^{2/n} - \chi^{2/n}}{\rho^{2/n} - 1} p_{0}$$
 (9)

$$\sigma_{\theta} = -\frac{\rho^{2/n} - \left(\frac{n-2}{n}\right) \chi^{2/n}}{\rho^{2/n} - 1} p_{0}$$
 (10)

$$\sigma_{z} = -\frac{\rho^{2/n} - \left(\frac{n-1}{n}\right)\chi^{2/n}}{\rho^{2/n} - 1} p_{0}$$
(11)

$$\overline{\sigma} = -\frac{\sqrt{3}}{n} \frac{\chi^{2/n}}{\rho^{2/n} - 1} p_0 \tag{12}$$

Assumption (4) is modified for negative strain rates assuming that compressive behavior is similar to tensile behavior. The resulting strain rates are

$$\dot{\epsilon}_{\mathbf{r}} = \frac{\sqrt{3}}{2} \operatorname{Be}^{-\Delta H/RT} \overline{\sigma}^{n}$$
 (13)

$$\dot{\epsilon}_{\theta} = -\frac{\sqrt{3}}{2} \operatorname{Be}^{-\Delta H/RT} \overline{\sigma}^{n} \tag{14}$$

$$\dot{\epsilon}_{\rm z} = 0$$
 (assumption (5)) (15)

$$\frac{\dot{\epsilon}}{\epsilon} = -B\overline{\sigma}^{n} e^{-\Delta H/RT} \qquad (assumption (4), compressive creep rate)$$
 (16)

Equations (13) to (15) relate the stresses and strain rates to functions of pressure, geometry, temperature, and materials properties for a given radius in the wall of a tube. The sign of the empirical exponential stress function $\overline{\sigma}^n$ is assumed to be positive.

DISCUSSION

Equations are presented for the stresses and strain rates in isothermal thick-walled tubes with static internal or external pressure that are applicable over a range of temperatures. Tube structures that are designed for long lifetimes at high temperatures may undergo creep deformation continuously. Design stresses must be selected so that creep rates are limited to very low secondary creep rates. If this is done, tertiary creep deformation will be avoided. The equations presented apply only for tubes undergoing secondary creep deformation.

The power law between stress and strain rate used in developing the equations contains a temperature correction term. Thus, the equations obtained are more general than previous work because they can be used to describe strain-rate data for a range of temperatures.

The equations presented constitute a closed-form solution to the problem of isothermal secondary creep in thick-walled tubes loaded with either internal or external pressure. When the material properties, constants, and tube geometry are substituted into the equations, the creep strain rates and stresses are completely described throughout the wall of the tube.

Fuel-pin cladding and heat-exchanger tubing are generally designed for thousands of hours of operation. The principal source of long-term deformation of these tubular structures will be secondary creep from pressures acting on the structures.

Some observations can be made by comparing the equations for stress and strain rate for external and internal pressure loading. The equations for strain rate are similar but of opposite sign. The equations for stresses $\sigma_{\mathbf{r}}$, σ_{θ} , and $\sigma_{\mathbf{z}}$ are different, while the equations for the equivalent stresses are the same but of opposite sign. If the internal pressure and the external pressure acting on a tube are equal, the net equivalent stress will be zero everywhere in the tube wall. Consequently, no creep will occur.

CONCLUDING REMARKS

Temperature-dependent strain-rate equations are presented for thick-walled tubes undergoing isothermal secondary creep from <u>internal</u> pressure. Temperature-dependent stress and strain-rate equations are provided for thick-walled tubes undergoing isothermal secondary creep from external pressure.

These equations are applicable to the analysis of stresses and long-term isothermal creep deformation of thick-walled tubular structures loaded by static internal or external pressures.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 10, 1971,
126-15.

APPENDIX - DERIVATION OF STRESS AND STRAIN-RATE EQUATIONS FOR THICK-WALLED TUBES AT CONSTANT TEMPERATURE

UNDERGOING SECONDARY CREEP

FROM INTERNAL PRESSURE

The derivations in this appendix are from the analytical precedure presented by Bailey in reference 1. The assumption that $\epsilon_z = 0$ is in agreement with experimental evidence. The assumption implies that planes perpendicular to the axis of a tube remain plane. This is the condition of plane strain.

Force equilibrium for cylindrical geometry in the absence of body force yields

$$\mathbf{r} \frac{\mathbf{d}\sigma_{\mathbf{r}}}{\mathbf{d}\mathbf{r}} = \sigma_{\theta} - \sigma_{\mathbf{r}} \tag{A1}$$

The strain displacement relations are

$$\epsilon_{\theta} = \frac{\mathbf{u}}{\mathbf{r}}$$

$$\epsilon_{\mathbf{r}} = \frac{d\mathbf{u}}{d\mathbf{r}}$$

Differentiating with respect to time yields

$$\begin{vmatrix}
\dot{\epsilon}_{\theta} = \frac{\dot{u}}{r} \\
\dot{\epsilon}_{r} = \frac{d\dot{u}}{dr}
\end{vmatrix}$$
(A2)

(where dots indicate d/dt). The compatibility equations for this case reduct to

$$\mathbf{r} \frac{d\dot{\epsilon}_{\theta}}{d\mathbf{r}} = \dot{\epsilon}_{\mathbf{r}} - \dot{\epsilon}_{\theta} \tag{A3}$$

Creep is a constant volume process, so that

$$\dot{\epsilon}_{\mathbf{r}} + \dot{\epsilon}_{\theta} + \dot{\epsilon}_{\mathbf{z}} = 0$$

Applying the assumption that $\dot{\epsilon}_z$ = 0, we have

$$\dot{\epsilon}_{\mathbf{r}} = -\dot{\epsilon}_{\theta} \tag{A4}$$

Substituting equation (A4) into (A3) yields

$$\frac{\mathrm{d}\dot{\epsilon}_{\theta}}{\dot{\epsilon}_{\theta}} = -\frac{2\mathrm{d}\mathbf{r}}{\mathbf{r}}$$

Upon integration, we have

$$\dot{\epsilon}_{\theta} = Cr^{-2} \tag{A5}$$

where the constant C is to be evaluated.

The assumption that the principal shear strain rates are proportional to the principal shear stresses can be expressed as follows:

$$\frac{\dot{\epsilon}_{\theta} - \dot{\epsilon}_{\mathbf{r}}}{\sigma_{\theta} - \sigma_{\mathbf{r}}} = \frac{\dot{\epsilon}_{\mathbf{r}} - \dot{\epsilon}_{\mathbf{z}}}{\sigma_{\mathbf{r}} - \sigma_{\mathbf{z}}} = \frac{\dot{\epsilon}_{\mathbf{z}} - \dot{\epsilon}_{\theta}}{\sigma_{\mathbf{z}} - \sigma_{\theta}}$$
(A6)

Substituting equation (A4) into (A6) yields

$$\sigma_{\mathbf{z}} = \frac{\sigma_{\theta} + \sigma_{\mathbf{r}}}{2} \tag{A7}$$

An expression for the equivalent stress $\overline{\sigma}$ is given by the distortion energy theory (also known as the von Mises yield criterion):

$$\overline{\sigma} = \frac{1}{\sqrt{2}} \left[(\sigma_{\theta} - \sigma_{\mathbf{r}})^2 + (\sigma_{\mathbf{r}} - \sigma_{\mathbf{z}})^2 + (\sigma_{\mathbf{z}} - \sigma_{\theta})^2 \right]^{1/2}$$
(A8)

Substituting for $\sigma_{\mathbf{z}}$ from equation (A7) gives

$$\overline{\sigma} = \frac{\sqrt{3}}{2} \left(\sigma_{\theta} - \sigma_{\mathbf{r}} \right) \tag{A9}$$

The equivalent strain rate $\stackrel{\cdot}{\overline{\epsilon}}$ corresponding to the equivalent stress is given by

$$\frac{\dot{\epsilon}}{\dot{\epsilon}} = \frac{\sqrt{2}}{3} \left[(\dot{\epsilon}_{r} - \dot{\epsilon}_{\theta})^{2} + (\dot{\epsilon}_{\theta} - \dot{\epsilon}_{z})^{2} + (\dot{\epsilon}_{z} - \dot{\epsilon}_{r})^{2} \right]^{1/2}$$
(A10)

Substituting for $\dot{\epsilon}_{\rm Z}$ = 0 and $\dot{\epsilon}_{\rm r}$ = $-\dot{\epsilon}_{\theta}$ reduces equation (A10) to

$$\frac{\dot{\epsilon}}{\dot{\epsilon}} = \frac{2}{\sqrt{3}} \dot{\epsilon}_{\theta} \tag{A11}$$

Make the assumption that experimental strain-rate data can be represented by

$$\frac{\dot{\epsilon}}{\epsilon} = B \overline{\sigma}^{n} e^{-\Delta H/RT}$$
 (A12)

Then

$$\frac{2}{\sqrt{3}} \dot{\epsilon}_{\theta} = B \left[\frac{\sqrt{3}}{2} (\sigma_{\theta} - \sigma_{r}) \right]^{n} e^{-\Delta H/RT}$$
(A12)

and from equations (A5) and (A1)

$$\left(\frac{2}{\sqrt{3}}\frac{C}{r^2}\right)^{1/n} = B^{1/n}e^{-\Delta H/nRT}\left(\frac{\sqrt{3}}{2}\right)r\frac{d\sigma_r}{dr}$$

$$d\sigma_{\mathbf{r}} = \left(\frac{2}{\sqrt{3}}\right)^{(n+1)/n} \left(\frac{Ce^{\Delta H/RT}}{Br^2}\right)^{1/n} \frac{d\mathbf{r}}{\mathbf{r}}$$
(A13)

Let

$$D = \left[\left(\frac{2}{\sqrt{3}} \right)^{n+1} \frac{Ce^{\Delta H/RT}}{B} \right]^{1/n}$$

$$d\sigma_{\mathbf{r}} = D \frac{d\mathbf{r}}{r^{(n+2)/n}}$$

Integration yields

$$\sigma_{\mathbf{r}} = D\left(-\frac{n}{2}\right)\frac{1}{r^{2}/n} + \mathbf{F}$$

The constants of integration C and F are evaluated using the boundary conditions. At r = b, $\sigma_{\bf r}$ =0

$$\sigma_{\mathbf{r}} = \mathbf{D}\left(-\frac{\mathbf{n}}{2}\right)\frac{1}{\mathbf{b}^{2/n}} + \mathbf{F} = 0$$

$$\mathbf{F} = \frac{\mathbf{Dn}}{2\mathbf{b}^{2/n}}$$

$$\sigma_{\mathbf{r}} = \frac{Dn}{2} \left(\frac{1}{b^{2/n}} - \frac{1}{r^{2/n}} \right)$$

At r = a, $\sigma_r = -p$

$$\sigma_{\mathbf{r}} = \left(\frac{\mathbf{D}\mathbf{n}}{2}\right) \left[\frac{1}{\mathbf{b}^{2/n}} - \frac{1}{\mathbf{a}^{2/n}}\right] = -\mathbf{p}$$

With D replaced

$$\sigma_{\mathbf{r}} = \frac{n}{2} \left[\left(\frac{2}{\sqrt{3}} \right)^{n+1} \frac{Ce^{\Delta H/RT}}{B} \right]^{1/n} \left(\frac{1}{b^{2/n}} - \frac{1}{a^{2/n}} \right) = -p$$

Then

$$C^{1/n} = -\frac{p}{\frac{n}{2} \left[\left(\frac{2}{\sqrt{3}} \right)^{n+1} \frac{e^{\Delta H/RT}}{B} \right]^{1/n} \left(\frac{1}{b^{2/n}} - \frac{1}{a^{2/n}} \right)}$$

Substituting for C and letting $\chi = b/r$ and $\rho = b/a$ give

$$\sigma_{\mathbf{r}} = -\frac{\chi^{2/n} - 1}{\rho^{2/n} - 1} p \tag{A15}$$

Differentiating with respect to r in equation (A15) gives

$$\frac{d\sigma_{\mathbf{r}}}{d\mathbf{r}} = \left(\frac{2}{n}\right)\left(\frac{1}{r}\right) \frac{\chi^{2/n}p}{\rho^{2/n}-1}$$

Substitution into the equilibrium equation (A1) yields the equation for σ_{θ} :

$$\sigma_{\theta} = \frac{1 - \left(\frac{n-2}{n}\right) \chi^{2/n}}{\rho^{2/n} - 1} p \tag{A16}$$

From equation (A7)

$$\sigma_{\mathbf{z}} = \frac{1}{2} (\sigma_{\mathbf{r}} + \sigma_{\theta})$$

$$= \frac{1}{2} \left(\frac{1 - \frac{n-2}{n} \chi^{2/n}}{\rho^{2/n} - 1} p - \frac{\chi^{2/n} - 1}{\rho^{2/n} - 1} p \right)$$

$$\sigma_{z} = \frac{1 - \frac{n-1}{n} \chi^{2/n}}{\rho^{2/n} - 1} p$$
 (A17)

From equation (A9)

$$\overline{\sigma} = \frac{\sqrt{3}}{n} \frac{\chi^{2/n}}{\rho^{2/n} - 1} p \tag{A18}$$

From equations (A4), (A11), and (A12)

$$\dot{\epsilon}_{\mathbf{r}} = -\frac{\sqrt{3}}{2} \, \mathbf{B} \overline{\sigma}^{\mathbf{n}} \mathbf{e}^{-\Delta \mathbf{H}/\mathbf{R} \mathbf{T}} \tag{A19}$$

$$\dot{\epsilon}_{\theta} = \frac{\sqrt{3}}{2} \, B \overline{\sigma}^{n} e^{-\Delta H/RT}$$
 (A20)

$$\dot{\epsilon}_{z} = 0$$
 (assumption (5)) (A21)

$$\frac{\dot{\epsilon}}{\dot{\epsilon}} = B \overline{\sigma}^{n} e^{-\Delta H/RT}$$
 (A22)

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